Aeroelastic Divergence of Swept-Forward Composite Wings Including Warping Restraint Effect

L. Librescu* and A. A. Khdeir†
Virginia Polytechnic Institute and State University, Blacksburg, Virginia

A powerful analytical approach to the divergence instability of laminated composite swept-forward wings is developed in the paper. The approach, based on the state space concept (used in conjunction with Jordan canonical form), enables one to solve exactly the equations governing the aeroelastic divergence of swept-forward composite wings, the warping restraint effect being incorporated into the analysis. The results obtained here emphasize the complex role played by the warping restraint effect in the divergence instability of swept-forward composite wings.

Nomenclature

a_o	= lift curve slope
a_1,a_2	= two speed parameters [Eqs. (5)]
A	= matrix defined by Eqs. (11) and (13)
AR	= wing aspect ratio
c	= wing chord measured perpendicular to the reference axis
C_{ij}	= stretching rigidities of the composite wing [Eq. (7a)]
D_{ij}	= bending rigidities of the composite wing [Eq. (7c)]
$\tilde{D}_{ij}$	= composite coupling stiffness rigidities [Eq. (6)]
e	= distance between spanwise reference axis and aerodynamic center line
G	= composite nondimensional torsional coupling parameter [Eq. (3b)]
K	= composite nondimensional bending coupling parameter [Eq. (3a)]
K_{ij}	= bending-stretching coupling rigidities [Eq (7b)]
ℓ	= wing semispan measured along the reference axis
q	= dynamic pressure ($\equiv 1/2\rho V^2$)
S	= warping rigidity [Eq. (4)]
V	= freestream velocity (parallel to the longitudinal axis of the airplane)
$X(\eta)$	= state space vector defined by Eqs. (11) and (13)
y	= distance along reference axis measured from the wing root
$Z,\tilde{Z}$	= bending deflection of the reference axis
η	$= y/\ell$
λ_r	= distinct eigenvalues of the matrix A
θ	= torsional deflection of wing sections about the reference axis
$\theta_{(j)}$	= orthotropicity angle of each lamina (j) with respect to a rearward normal to the reference axis and measured in the counterclockwise direction
Λ	= sweep angle of the reference axis (positive for swept back)

Subscripts and superscripts

D	= divergence quantities
$(\cdot)^{(n)}$	$= \partial^n(\ )/\partial\eta^n$
$(\ )^*$	= divergence quantity determined by including warping restraint effect (WRE)

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*Professor, Department of Engineering Science and Mechanics.
†Assistant Professor, Department of Engineering Science and Mechanics.

Introduction

DURING the last two decades, a great deal of research activity has been devoted to the development of a new concept in the design of aeronautical and space structures. This concept, referred to as aeroelastic tailoring, is based on the employment of the exotic properties of advanced filamentary composite materials in order to enhance their aeroelastic response characteristics.

Developed in the past in the analysis of the supersonic panel flutter problem (see, for example, Ref. 1), it has reached in the last decade new promising areas of applicability that concern the aeroelastic design of aircraft wings and of other devices experiencing aeroelastic instability phenomena (such as turbine and helicopter blades). For a comprehensive survey paper reviewing in depth the state of the art of this problem, the reader is referred to Ref. 2. A most spectacular application of this concept is constituted by the forward swept wing (FSW) aircraft. As is well known,[3] due to the low divergence instability speed experienced by the metallic FSW aircraft, its employment as a possible option was completely eliminated for a long time. However, the studies prompted by Krone,[4] continued by Weisshaar,[5–7] and followed in a series of other theoretical and experimental works[7–15] have revealed that a composite FSW can be aeroelastically tailored to overcome this adverse instability phenomenon.

In this connection, it should be stressed that with a very few exceptions,[13,15] the free-warping (FW) model for the wing twist has been unanimously adopted in the treatment of this problem. In spite of this, the results obtained in Refs. 15 and 16 related to the divergence instability, as well as the ones derived in Refs. 17–22, have revealed the great importance of the axial warping restraint effect (WRE) on the behavior of cantilevered-type structures. In addition, as it was shown in Ref. 15: 1) in the case of anisotropic composite FSW, the warping restraint effect could also affect high-aspect-ratio wings, and 2) WRE could result not only in the increase of the critical divergence speed with respect to its FW counterpart but also in its decrease. (In other terms, the classical FW model could yield nonconservative results from the divergence instability point of view.)

This paper is devoted to the analysis of the divergence instability of the swept-forward composite wing by also incorporating WRE. Toward this goal, a powerful method based on the state space concept and used in conjunction with the Jordan canonical form is applied. In contrast to the usual way of using it in the control theory, here this method is applied in the spatial domain.

Alternative approaches to the divergence instability of swept-forward wings (restricted to the free-warping case) formulated in terms of a hybrid state vector and using a matrix integration procedure can be found in Refs. 10, 22, and 23. However, the approach developed in this paper is exact, and no other assumptions were used beyond the ones stipulated in the modeling of the problem.

Preliminary Basic Assumptions (Aeroelastic) Governing Equations

The anisotropic laminated plate-beam model and a corrected strip theory aerodynamics (in the sense of Ref. 5) are used to analyze the divergence instability of swept cantilevered wing structures.

The constituent laminas are characterized by different orthotropicity angles and by different material and thickness properties. The interface plane between the contiguous layers r and $r+1$ ($1<r<N$, where N denotes the total number of constituent layers) will be selected as the reference plane of the composite wing structure. As in Refs. 5 and 6, we also shall postulate the existence of a reference axis (RA), coinciding with the y axis and located in the reference plane of the box-beam and at middistance between its front and rear edges.

The angle of sweep (considering positive for swept-back and negative for swept-forward wings) is measured in the x-y plane of the wing from the direction normal to the airflow to the reference axis. The wing is considered clamped normal to this RA, its effective length ℓ being measured along this axis. All parameters associated with the wing sections, such as chord c, location of the aerodynamic center, etc., are based on sections normal to the RA (see Fig. 1).

The material of each constituent lamina is assumed to be orthotropic, where the orthotropicity angle $\theta_{(j)}$ of each lamina is measured in the counterclockwise direction, starting from the rearward normal to the y axis. In addition to these assumptions, we also postulate that the chordwise deformation, as well as the wing distortions, are negligibly small. In light of these assumptions, the equations governing the static aeroelastic equilibrium of uniform cross-section composite swept wings may be written in nondimensional form as

$$\tilde{Z}^{\mathrm{IV}} - K\theta''' = a_1(\theta - \tilde{Z}' \tan\Lambda) \tag{1a}$$

$$S\theta^{\mathrm{IV}} - \theta'' + G\tilde{Z}''' = a_2(\theta - \tilde{Z}' \tan\Lambda) \tag{1b}$$

while the associated boundary conditions (BC) are

$$\tilde{Z} = \tilde{Z}' = \theta' = \theta = 0$$

at $\eta = 0$ and

$$\tilde{Z}'' - K\theta' = 0 \tag{2a}$$

$$\tilde{Z}''' - K\theta'' = 0 \tag{2b}$$

$$-S\theta''' + \theta' - G\tilde{Z}'' = 0 \tag{2c}$$

$$S\theta'' = 0 \tag{2d}$$

at $\eta = 1$. (For a derivation of these equations, see Ref. 15.) In Eqs. (1) and (2), $\tilde{Z}(\equiv Z/\ell)$ and θ denote the nondimensional deflection of the RA and the twist about the RA, respectively. The twisting angle θ is not to be confused with the orthotropicity angle $\theta_{(j)}$. The equations

$$K \equiv \tilde{D}_{26}/\tilde{D}_{22} \tag{3a}$$

$$G \equiv \tilde{D}_{26}/\tilde{D}_{66} \tag{3b}$$

denote the nondimensional bending and torsional coupling parameters, respectively;

$$S \equiv (c^3/12\ell^2)(D_{22}/\tilde{D}_{66}) \tag{4}$$

denotes the warping rigidity;

$$a_1 \equiv q_n c\ell^3 a_o/\tilde{D}_{22} \tag{5a}$$

$$a_2 \equiv q_n ec\ell^2 a_o/\tilde{D}_{66} \tag{5b}$$

denote two speed parameters associated with the bending and torsional degrees of freedom, respectively [connected by $a_2 = \alpha a_1$, where $\alpha \equiv eG/(\ell K)$]; $q_n(\equiv \rho_o V_n^2/2) = q\cos^2\Lambda$ denotes the dynamic pressure component normal to the leading edge, a_o the lift curve slope coefficient, and e the offset between the aerodynamic and reference axes.

Here[15]

$$\tilde{D}_{22} \equiv c(D_{22} - K_{22}^2/C_{22}) \tag{6a}$$

$$\tilde{D}_{26} \equiv 2c(D_{26} - K_{22}K_{26}/C_{22}) \tag{6b}$$

$$\tilde{D}_{66} \equiv 4c(D_{66} - K_{26}^2/C_{22}) \tag{6c}$$

stand for the coupling stiffness parameters, where

$$C_{ij} = \sum_{k=1}^{N} (\bar{Q}_{ij})_{(k)}[h_{(k)} - h_{(k-1)}] \tag{7a}$$

$$K_{ij} = \frac{1}{2}\left\{\sum_{k=1}^{r} (\bar{Q}_{ij})_{(k)}[h_{(k)}^2 - h_{(k-1)}^2] - \sum_{k=r+1}^{N} (\bar{Q}_{ij})_{(k)}[h_{(k)}^2 - h_{(k-1)}^2]\right\} \tag{7b}$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} (\bar{Q}_{ij})_{(k)}[h_{(k)}^3 - h_{(k-1)}^3] \quad (i,j = 1,2,6) \tag{7c}$$

define the anisotropic stiffness in stretching, bending-stretching, and bending, respectively.[1] In Eqs. (1) and (2), the derivatives with respect to the nondimensional spanwise coordinate $\eta(\equiv y/\ell)$ are denoted as $(\cdot)^{(n)} \equiv \partial^n(\cdot)/\partial\eta^n$. For the sake of completion, the free-warping counterparts of the field equations, Eqs. (1) and (2), are recorded here. They read[7,15]

$$\tilde{Z}^{\mathrm{IV}} - K\theta''' = a_1(\theta - \tilde{Z}' \tan\Lambda) \tag{8a}$$

$$G\tilde{Z}''' - \theta'' = a_2(\theta - \tilde{Z}' \tan\Lambda) \tag{8b}$$

while the BC are

$$\tilde{Z} = \tilde{Z}' = \theta = 0 \tag{8c}$$

at $\eta = 0$ and

$$\tilde{Z}'' - K\theta' = 0 \tag{9a}$$

$$\tilde{Z}''' - K\theta'' = 0 \tag{9b}$$

$$\theta' - G\tilde{Z}'' = 0 \tag{9c}$$

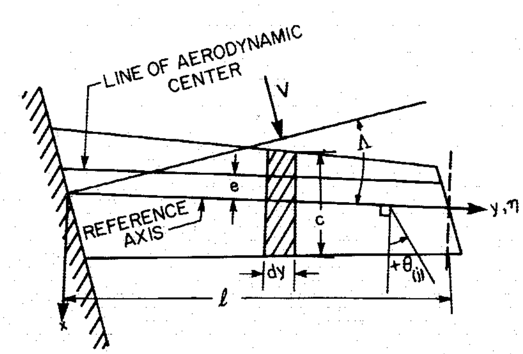

Fig. 1 Geometry of the swept wing.

at $\eta = 1$. The foregoing exhibited equations will be used in the analysis of the divergence instability of swept-forward composite wing structures.

Solution Procedure

The state space concept in the spatial domain will be used to analyze the divergence instability of swept-forward wings. According to this concept, the governing equations belonging both to the warping restraint case and to its free-warping counterpart may be converted in matrix form as

$$X' = AX \tag{10}$$

In an explicit form and for the warping restraint case, Eqs. (1) as converted reads

$$\begin{bmatrix} X_1'(\eta) \\ X_2'(\eta) \\ X_3'(\eta) \\ X_4'(\eta) \\ X_5'(\eta) \\ X_6'(\eta) \\ X_7'(\eta) \\ X_8'(\eta) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & e_2 & 0 & 0 & e_3 & 0 & 0 & e_1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & e_6 & 0 & e_4 & e_7 & 0 & e_5 & 0 \end{bmatrix} \begin{bmatrix} X_1(\eta) \\ X_2(\eta) \\ X_3(\eta) \\ X_4(\eta) \\ X_5(\eta) \\ X_6(\eta) \\ X_7(\eta) \\ X_8(\eta) \end{bmatrix} \tag{11}$$

where $X_1 = \tilde{Z}(\eta)$ and $X_5 \equiv \theta(\eta)$ and

$$e_1 \equiv K,\ e_2 \equiv -a_1 \tan\Lambda,\ e_3 \equiv a_1,\ e_4 \equiv -G/S,\ e_5 \equiv 1/S$$
$$e_6 \equiv -(a_2/S)\tan\Lambda,\ e_7 \equiv a_2/S \tag{12}$$

In the case of free warping, the counterpart of Eq. (11) is

$$\begin{bmatrix} X_1'(\eta) \\ X_2'(\eta) \\ X_3'(\eta) \\ X_4'(\eta) \\ X_5'(\eta) \\ X_6'(\eta) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & e_2 & e_1 & 0 & e_4 & e_3 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & e_6 & 0 & e_5 & e_7 & 0 \end{bmatrix} \begin{bmatrix} X_1(\eta) \\ X_2(\eta) \\ X_3(\eta) \\ X_4(\eta) \\ X_5(\eta) \\ X_6(\eta) \end{bmatrix} \tag{13}$$

where $X_1 \equiv \tilde{Z}(\eta)$; $X_5 \equiv \theta(\eta)$, whereas

$$e_1 \equiv Ka_2 \tan\Lambda/(1-KG),\ e_2 \equiv -a_1 \tan\Lambda/(1-KG)$$
$$e_3 \equiv -Ka_2/(1-KG),\ e_4 \equiv a_1/(1-KG)$$
$$e_5 \equiv G,\ e_6 \equiv a_2 \tan\Lambda,\ e_7 = -a_2 \tag{14}$$

The reader is reminded of the result obtained in Ref. 6, where it was shown that the cross-coupling stiffness parameter KG fulfills the condition $KG < 1$.

A formal solution to Eq. (10) is given by

$$X(\eta) = e^{A\eta} \boldsymbol{K} \tag{15}$$

where $\boldsymbol{K}$ is a constant column vector connected with the boundary conditions.

Having in view that the matrix A has in both cases an eigenvalue with multiplicity 3 (the remaining ones being distinct), the matrix $e^{A\eta}$ in Eq. (15) may be represented in terms of the Jordan canonical form as

$$e^{A\eta} = Pe^{J\eta}P^{-1} \tag{16}$$

Here, $e^{J\eta}$ is block diagonal, the Jordan matrix J being characterized by its quasidiagonal form

$$J = \mathrm{diag}(J_1, J_2, \ldots, J_n) \tag{17}$$

and by the special structure of the diagonal blocks. In both cases, the block J_1 associated with the eigenvalue λ_1 of multiplicity 3 assumes the form

$$J_1 = \begin{bmatrix} \lambda_1 & 1 & 0 \\ & \lambda_1 & 1 \\ 0 & & \lambda_1 \end{bmatrix} \tag{18}$$

The remaining blocks J_r are expressed as $J_r = [\lambda_r]$, with λ_r denoting the distinct eigenvalues of the matrix A, where, corresponding to the warping restraint and free-warping instances, r assumes the range $r = \overline{2,6}$ and $r = \overline{2,4}$, respectively. Concerning the transformation matrix P intervening in Eq. (16), it should contain the eigenvectors and the generalized eigenvectors. Its determination follows the general procedure presented in Ref. 26.

Having in view Eqs. (15) and (16), the solution X is modified as

$$X(\eta) = Pe^{J\eta}P^{-1}\boldsymbol{K} \tag{19}$$

Equation (19) considered in conjunction with the BC [Eqs. (2) and (9) associated with WR and WF instances, respectively] yields an homogeneous system of equations

$$M_{ij}K_j = 0 \tag{20}$$

where $i, j = \overline{1,8}$ and $\overline{1,6}$ for WR and FW, respectively.

The argument of nontriviality of the solution in Eq. (20) results in the condition of divergence expressed in determinantal form. The divergence condition thus is given by the vanishing of the determinant of the matrix M_{ij}. The minimum positive root a_1 of the obtained equation of instability constitutes the divergence instability speed $(a_1)_D$.

Numerical Applications

Several composite wing-structure configurations are considered in the numerical analysis of the divergence instability. The materials of the envisaged structures are as follows:

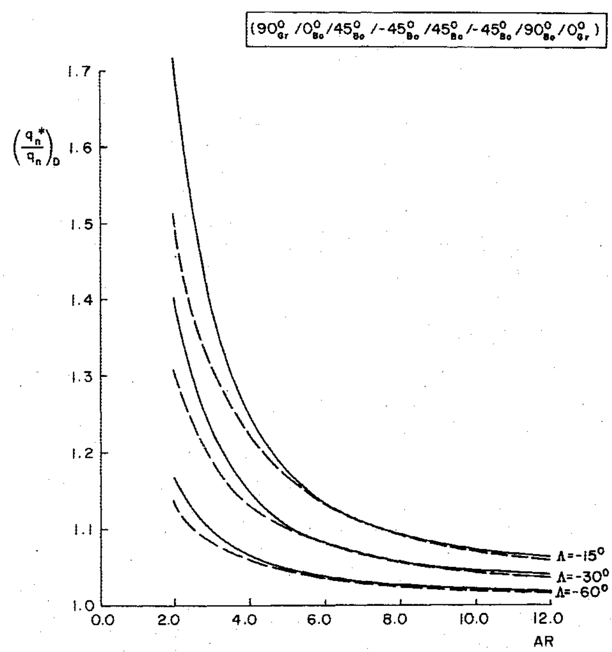

Fig. 2 Depiction of $(q_n^*/q_n)_D$ vs AR for the wing structure labeled a and for the various sweep angles. Depicted results concern the exact solution (identified by the interrupted lines) and the approximate one[15] (identified by the solid lines).

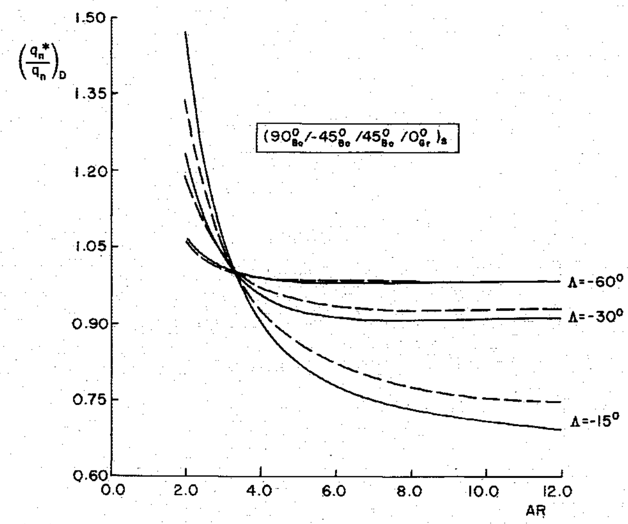

Fig. 3 Depiction of $(q_n^*/q_n)_D$ vs AR for the wing structure labeled b and for various sweep angles. Solid lines identify the approximate solution[15] and the interrupted ones the exact solution (developed in this paper).

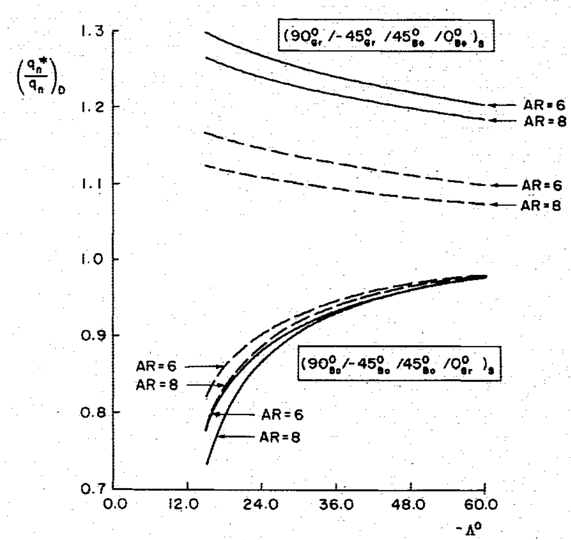

Fig. 4 Depiction of $(q_n^*/q_n)_D$ vs Λ deg for the wing structures labeled c and d. Results concern the exact solution obtained in this paper (identified by the interrupted lines) and the approximate one[15] (identified by solid lines).

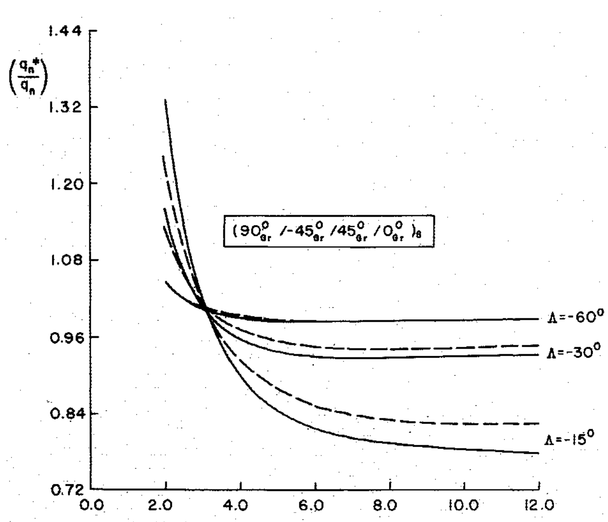

Fig. 5 Depiction of $(q_n^*/q_q)_D$ vs AR for the wing structure labeled d. Solid lines identify the approximate solution[15] and the interrupted ones the exact solution (obtained by using the present approach).

Table 1 Variation of $(a_1)_D$ for the free-warping case of the wing structure labeled e

	$\theta = 30$ deg	$\theta = 60$ deg	$\theta = 120$ deg	$\theta = 150$ deg
		$\Lambda = 0$ deg		
$(a_1)_D$: Ref. 15	17.165	2.405		
$(a_1)_D$: Refs. 6 and 7	16.868	2.359	No divergence	No divergence
$(a_1)_D$: based on the present method	16.822	2.351	—	—
		$\Lambda = -30$ deg		
Ref. 15	5.239	1.355	No	13.130
Refs. 6 and 7	5.166	1.331	divergence	12.905
Present method	5.156	1.327	—	12.961
		$\Lambda = -60$ deg		
Ref. 15	2.192	0.724	1.963	3.015
Refs. 6 and 7	2.164	0.711	1.951	2.987
Present method	2.160	0.709	1.954	2.987

1) Boron-epoxy (Bo) with the characteristics

$$E_{11} = 30 \times 10^6 \text{ psi}, E_{22} = 3 \times 10^6 \text{ psi}$$

$$G_{12} = 1 \times 10^6 \text{ psi}, \nu_{12} = 0.3$$

where the thickness t of each ply is $t = 0.1$ in.

2) Graphite-epoxy (Gr) with the characteristics

$$E_{11} = 30 \times 10^6 \text{ psi}, E_{22} = 0.7 \times 10^6 \text{ psi}$$

$$G_{12} = 0.375 \times 10^6 \text{ psi}, \nu_{12} = 0.25, t = 0.005 \text{ in.}$$

The structures themselves are defined and labeled as follows:

Structure a): Hybrid nonsymmetric

$$(90^\circ{}_{Gr}/0^\circ{}_{Bo}/45^\circ{}_{Bo}/-45^\circ{}_{Bo}/45^\circ{}_{Bo}/-45^\circ{}_{Bo}/90^\circ{}_{Bo}/0^\circ{}_{Gr})$$

Structure b): Hybrid symmetric

$$(90^\circ{}_{Bo}/-45^\circ{}_{Bo}/45^\circ{}_{Bo}/0^\circ{}_{Gr})_s$$

Structure c): Hybrid symmetric

$$(90^\circ{}_{Gr}/-45^\circ{}_{Gr}/45^\circ{}_{Bo}/0^\circ{}_{Bo})_s$$

Structure d): Symmetric graphite-epoxy

$$(90^\circ/-45^\circ/45^\circ/0^\circ)_s$$

Structure e): $(\theta^\circ_{Bo})_{20}$

The boron-epoxy material used in this case corresponds to the characteristics given in Ref. 6.

In all of these numerical applications it was considered that $e = 0.1c$. Figures 2–5 display the variation of $(q_n^*/q_n)_D$ vs AR and Λ, where $(q_n^*)_D$ and $(q_n)_D$ denote q_n of divergence determined for WR and FW instances, respectively. Table 1 exhibits several solutions for $(a_1)_D$ obtained from the FW model, for the wing structure labeled e and compared with the exact solution obtained in Refs. 6 and 7.

Conclusions

The results obtained on the basis of this approach show that:

1) WRE could not only be beneficial (as the earlier results obtained for the case of metallic wings in Ref. 16 have revealed) but also results in a reduction of the divergence speed, with

respect to its free-warping counterpart, as can be seen from Figs. 3–5.

2) WRE could be significant in the case of large aspect ratio wings.

These facts, previously revealed in an approximate way in Ref. 15, enforce the conclusions that, due to its complex and significant role (of a beneficial or detrimental nature), WRE is to be considered whenever the divergence instability of composite swept-forward wings is investigated. In addition, these results compared with their approximate counterparts obtained in Ref. 15 show an excellent agreement. However, in very few instances numerical differences with respect to the exact solution may occur. They could be due to the approximate selection of modal functions in Ref. 15.

In spite of the very few cases when such differences occur, the trend of the influence of WRE is always accurately revealed by the approximate solution. In addition, the results indicate that the exact solution (derived either on the basis of the method developed here or on the one used in Ref. 7) and its approximate counterpart obtained for the WF instance are always in excellent agreement, for both unswept and swept wing structures.

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